



Terminal guidance laws of missile based on ISMC and NDOB with impact angle constraint



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ARTICLE INFO

Article history:

Received 5 March 2012

Received in revised form 2 June 2013

Accepted 5 September 2013

Available online 18 September 2013

Keywords:

Missile

Terminal guidance law

Integral sliding mode control

Nonlinear disturbance observer

Finite-time control

ABSTRACT

The terminal guidance problem for missiles intercepting maneuvering targets with terminal impact angle constraints is investigated. Regarding the target acceleration as an unknown bounded disturbance, novel guidance laws based on integral sliding mode control (ISMC) method technique are developed. The first one is a linear integral sliding mode (ISM) guidance law, which can guarantee the line-of-sight (LOS) angular rate and the LOS angle asymptotical convergence with infinite time. To further improve the convergence characteristics of guidance system, a nonlinear ISM guidance law is developed, which guarantees the LOS angular rate and LOS angle finite-time convergence characteristics. However, to guarantee the guidance system has a good performance for dealing with target acceleration, the switch gains of both linear and nonlinear ISM guidance laws need to be chosen larger than the bound of the target acceleration. It will lead to chattering problem. To reduce the chattering phenomenon and improve the performance of system, nonlinear disturbance observer (NDOB) is employed to estimate the target acceleration. The estimated acceleration is used to compensate to actual target acceleration. Then, two novel composite guidance laws combining linear and nonlinear ISM guidance laws with feedforward compensation terms based on NDOB are developed. Finally, simulation comparison results are provided to demonstrate the effectiveness of the proposed methods.

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1. Introduction

The objective of a missile guidance law is to provide acceleration commands to the missile autopilot that produce a minimum miss distance with respect to a given target [32]. For the past decades, proportional navigation (PN) guidance law is the most popular and widely used guidance law in the practical missile interception engagements because of its ease of implementation and high efficiency [19]. The PN guidance law is first introduced by [44] where the target moves on a straight course and a plane curved course are discussed, respectively. The principle of PN guidance law is that the commanded missile normal acceleration is proportional to the line-of-sight (LOS) angular rate [4]. It is well known that PN guidance law is the optimal guidance law with a navigation ratio $N = 3$, and it can result in successful interceptions under a wide range of engagement conditions [42]. Then many kinds of modified PN guidance laws are developed in the literature [12,24,43]. However, the PN and modified PN guidance laws are more applicable for the task of intercepting a nonmaneuvering target or a weakly maneuvering target. In practice, target acceleration can change rapidly. For intercepting a target with powerful maneuver-

ing capability, a significant miss distance may be resulted by PN or modified PN guidance laws [46]. To obtain a small miss distance, the guidance laws based on modern control theory are proposed, such as, optimal guidance law [37], differential game guidance law [26], nonlinear H_∞ guidance law [38], Lyapunov-based guidance law [17], L_2 gain-based guidance law [48], adaptive guidance law [10], sliding mode guidance law, etc.

It is well known that sliding mode control (SMC) is an efficient control method, not only because it has ability to deal with nonlinear perturbations, external disturbances and parametric uncertainties, which make the system have a good robust and disturbance rejection performance, but also because the discontinuous terms may allow the system states to have more quickly convergence characteristic than continue control laws would do [34]. In the guidance law design problem, the LOS angular rate is commonly used to define the sliding mode manifold because the physical principle is to attain a collision triangle [27]. The zero-effort miss distance is another candidate for designing the sliding mode manifold [45]. Considering the target acceleration as a bounded uncertainty, Moon et al. proposed a sliding mode guidance law [22]. The target acceleration is suppressed by the target acceleration bound. Shima et al. derived a sliding mode guidance law for an integrated missile autopilot and guidance law design problem with the zero-effort miss distance as sliding mode manifold [27].

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Lin et al. proposed an adaptive fuzzy sliding mode guidance law to force the missile to move along the instantaneous line of sight [20]. By the fuzzy rules learning online, the guidance law adjusts the parameters of the fuzzy rules and the target acceleration bound. Considering the flight dynamics of missile and actuator time delay, Brierley et al. proposed a sliding mode guidance law with the difference between the LOS angular rate and steering fin angle as the sliding mode manifold [5]. Zhou et al. proposed an adaptive sliding mode guidance law with the product of relative distance and LOS angular rate as the sliding mode manifold [47]. Ebrahimi et al. proposed an optimal sliding mode guidance law with the missile terminal velocity constraint by selecting the control energy as the optimization criterion and the combination of zero-effort miss distance and zero-effort velocity as the sliding mode manifold [11]. Alexander et al. proposed a sliding mode control guidance law for integrated guidance and control system using a sliding mode observers to estimate the target states [1]. All above SMC guidance laws have the ability to deal with the target acceleration.

The conventional SMC is powerful in controlling nonlinear systems, but, when the system states have not yet reached the sliding mode manifold, it is sensitive to parameter variation and external disturbance [7,35]. In order to improve the robustness in reaching phase, integral sliding mode control (ISMC) was proposed by Utkin et al. [35]. Compared with the conventional SMC, the integral sliding mode (ISM) dynamic starts from the initial time instance. Therefore, the reaching phase is eliminated and robustness of the system can be guaranteed throughout the entire response of the system. Hence, the performance of conventional SMC can be improved by introducing ISMC. Till now, ISMC schemes for linear and nonlinear systems have been studied, e.g., the design methods for ISM manifold [6,7], higher-order ISMC methods [16,49], and applied in many control fields, e.g., induction motor [2], five-degree-of-freedom active magnetic bearing system [21], omnidirectional mobile robot [15], etc. The integral action of ISMC can improve the reaching phase, at the same time, it may bring overshoot and lengthen the settling time. To overcome this problem, finite-time control strategy and NDOB technique are employed in this paper.

Actually, the missile guidance problem considered in most literatures are solved by the asymptotic stability analysis which implies that the LOS angular rate converges to zero with infinite settling time. It is well known that finite-time stabilization of dynamical systems may give rise to a better disturbance attenuation besides fast convergence to the origin [3]. Finite-time control strategy for the guidance systems have also been studied. SMC guidance laws with finite-time convergence were proposed by Zhou et al. [46]. The proposed guidance scheme involved a continuous homogeneous function to ensure finite-time convergence of the LOS angular rate. By considering the dynamics of a missile's autopilot as a first-order lag, Sun et al. proposed a SMC guidance law with finite time convergence [33]. Shtessel et al. proposed a composite SMC guidance law with finite time convergence characteristics for a missile guidance system against target performing evasive maneuvers [29] and for a two-loop integration of guidance and flight control systems with dynamic uncertainty [30].

For the SMC problem, there exists an unavoidable application problem-chattering. To alleviate chattering phenomenon, one of the solution is to employ a saturation function instead of signum function in the control input [36]. However, to do this, the disturbance rejection performance is sacrificed to some extent. Another efficient method for alleviating the chattering problem is to employ a disturbance observer (DOB) to estimate the disturbances. The disturbance estimation is used for compensation. The disturbance observer technique is first proposed in [23], and up to now, DOB-based control schemes for linear and nonlinear systems have been studied and applied in many control fields, such as non-smooth observer [18], robotic systems [9], grinding systems [39],

nonlinear magnetic levitation suspension system [40], mismatched uncertainties [41], general systems [8], etc.

When using a missile to intercept a target, the impact geometry is often important. The lethality of warheads can be increased at a specific impact angle. Thus, the perfect guidance law not only can guarantee the missile shortens the miss distance, but also can guarantee the missile has an appropriate attitude to hit target. Shin et al. proposed a guidance law by solving the two-point boundary value problem with impact angle constraint [28]. Ryoo et al. proposed energy optimal guidance laws with impact angle constraint for arbitrary missile dynamics [25]. Song et al. proposed a time optimal guidance law in the case of constrained missile maneuverability and a first-order-lag missile autopilot [31]. Harl et al. proposed a second-order sliding mode guidance law with terminal constraints of impact angle and impact time using a backstepping concept with the difference between LOS angle and desired LOS angle as sliding mode manifold [13]. Ratnoo et al. proposed a PN guidance law against a nonstationary nonmaneuvering target with impact angle constraint [24].

In this paper, a more interesting second-order nonlinear missile guidance problem with impact angle constraint will be considered, and novel guidance laws based on ISMC method and NDOB technique are proposed. A linear ISMC method is employed for asymptotically stable strategy, and, a nonlinear ISMC method is employed for finite-time stable strategy. The NDOB is used for estimating the target acceleration. The proposed guidance laws is designed with no information on the target acceleration, and, they can yield excellent performance and desirable robustness properties. First of all, a linear ISM guidance law based on a linear ISM manifold inspired by Zong [49] is designed by using Lyapunov theory under the assumption that the target acceleration is bounded by a constant upper bound. The law guarantees the guidance system asymptotically stable. To further improve the convergence characteristics of guidance system, a nonlinear ISM guidance law based on a nonlinear ISM manifold is proposed by using finite-time Lyapunov theory. The nonlinear law guarantees the guidance system finite-time stable. However, to guarantee the guidance system has a good performance for dealing with target acceleration, the switching gain needs to be chosen larger than the bound of the target acceleration. It will lead to chattering problem. Then, to reduce the chattering phenomenon and improve the performance of system, an efficient disturbance estimation technique, NDOB, is employed to estimate the target acceleration. After disturbance compensation based on NDOB, the switching gain only needs to be larger than the bound of the disturbance compensation error which is usually much smaller than that of the target acceleration. Hence, the chattering will be reduced while the disturbance rejection performance of the guidance system can be maintained. Then, novel composite guidance laws combination of ISMC method and NDOB technique are developed.

The rest of this paper is organized as follows. In Section 2, some preliminaries are briefly outlined. The main results are presented in Sections 3 and 4, respectively. In Section 3, linear and nonlinear ISM guidance laws are proposed. In Section 4, two composite guidance laws are developed combining the ISMC method with NDOB technique. To verify the effectiveness of the proposed guidance laws, simulation results are provided in Section 5. Finally, conclusions are drawn in Section 6.

2. Problem formulation

This section presents the mathematic model derivation of the guidance system for missile intercepting target. The geometry of planar interception is depicted in Fig. 1. We denote the missile and target by subscripts m and t , respectively. For the purpose of guidance law design, missile and target are assumed to be point

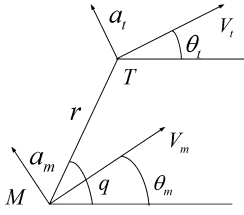


Fig. 1. Planar interception geometry.

masses, and only kinematics are considered. The corresponding equations of guidance system [26] are given by

$$r\dot{q} = V_m \sin(q - \theta_m) - V_t \sin(q - \theta_t), \quad (1)$$

$$\dot{r} = -V_m \cos(q - \theta_m) + V_t \cos(q - \theta_t), \quad (2)$$

$$\dot{\theta}_m = a_m / V_m, \quad (3)$$

$$\dot{\theta}_t = a_t / V_t, \quad (4)$$

where r and \dot{r} are the relative distance and the relative velocity between missile and target, q and \dot{q} are the LOS angle and LOS angular rate between missile and target, V_m and V_t are the missile velocity and the target velocity, θ_m and θ_t are the flight-path angle of missile and target, a_m and a_t are the missile lateral acceleration normal to missile velocity and the target lateral acceleration normal to target velocity, respectively.

Differential Eq. (1) with respect to time, yields

$$\begin{aligned} r\ddot{q} = & -2\dot{r}\dot{q} - a_m \cos(q - \theta_m) + \dot{V}_m \sin(q - \theta_m) \\ & + a_t \cos(q - \theta_t) - \dot{V}_t \sin(q - \theta_t). \end{aligned} \quad (5)$$

The design objects here are to develop guidance laws such that the guidance law can not only guarantee the missile hits the target with zero miss distance, but also guarantee the missile has an appropriate attitude to intercept the maneuvering target. That is to design the guidance laws a_m such that the LOS angular rate \dot{q} will converge to zero, and the LOS angle q will converge to a desired LOS angle. The main results will be shown in the following sections.

In this paper, the signals r , \dot{r} , q , \dot{q} and θ_m are assumed to be measurable. Let q_d be as the desired LOS angle. Let q_e be the LOS angle error, i.e., $q_e = q - q_d$. Defining $x_1 = q_e$, $x_2 = \dot{q}_e$, Eq. (5) can be written as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{1}{r} (-2\dot{r}x_2 - a_m \cos(q - \theta_m) + \dot{V}_m \sin(q - \theta_m) \\ \quad + a_t \cos(q - \theta_t) - \dot{V}_t \sin(q - \theta_t)). \end{cases} \quad (6)$$

Assumption 1. The target lateral acceleration $a_t(t)$ and longitudinal acceleration $\dot{V}_t(t)$ are assumed to be bounded and satisfy $|a_t(t)| \leq d_1$, $|\dot{V}_t(t)| \leq d_2$, for all $t \geq 0$, where d_1 and d_2 are the known bound of the target accelerations.

Assumption 2. During the time horizon of the guidance process, it has $\dot{r}(t) < 0$, $0 < r(t) < r(0)$, $t > 0$ [46]. The missile intercepting target by impact ("hit-to-kill") happens when $r \neq 0$ but belongs to the interval $r^0 \in [r_{\min}, r_{\max}] = [0.1, 0.25]$ m [30].

Before giving the guidance laws design, some results on the finite time stability are needed. Firstly, the definition of finite-time stability is presented as follows.

Definition 1. (See [3].) Consider a nonlinear system in the form

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in \mathbb{R}^n, \quad (7)$$

where $f(x) : U_0 \rightarrow \mathbb{R}^n$ is continuous on an open neighborhood U_0 of the origin $x = 0$. The equilibrium $x = 0$ of the system is (locally) finite-time stable if the following two conditions are satisfied: (i) It is asymptotically stable, in U , an open neighborhood of the origin $x = 0$ with $U \subseteq U_0 \subset \mathbb{R}^n$. (ii) It is finite-time convergent in U , that is, for any initial condition $x_0 \in U \setminus \{0\}$, there is a settling time $T > 0$, dependent on x_0 , such that every solution $x(t, x_0)$ of system (7) is defined with $x(t, x_0) \in U \setminus \{0\}$ for $t \in [0, T)$ and satisfies $\lim_{t \rightarrow T(x_0)} x(t, x_0) = 0$ and $x(t, x_0) = 0$, if $t \geq T(x_0)$. Moreover, if $U = \mathbb{R}^n$, the origin $x = 0$ is globally finite-time stable.

The following lemma will be used in the subsequent guidance laws development and analysis.

Lemma 1. (See [3].) Consider the nonlinear system described by Eq. (7). Suppose there exists a continuous function $V(x) : U \rightarrow \mathbb{R}$ such that:

- (i) $V(x)$ is positive definite.
- (ii) There exist real numbers $c > 0$ and $\alpha \in (0, 1)$ and an open neighborhood $U_0 \subset U$ of the origin such that $\dot{V}(x) \leq -cV^\alpha(x)$, $x \in U_0 \setminus \{0\}$.

Then, the origin is a finite-time stable equilibrium of system (7). Moreover, if T is the settling time, then $T(x) \leq \frac{1}{c(1-\alpha)} V^{1-\alpha}(x)$ for all x in some open neighborhood of the origin. If $U = U_0 = \mathbb{R}^n$, the origin is a globally finite-time stable equilibrium of system (7).

3. Guidance law based on integral sliding mode control

3.1. Guidance law based on linear integral sliding mode control

In this section, a linear ISM guidance law is presented for the terminal guidance system described by Eqs. (1)–(4) and (6). Let us give a brief description of the design method. Firstly, a linear ISM manifold inspired by Zong [49] is designed. Secondly, a linear ISM guidance law based on the linear ISM manifold is developed, and then it is proved that the obtained guidance law can guarantee that the LOS angular rate asymptotically converges to zero, and guarantee that the LOS angle asymptotically converges to the desired LOS angle.

Considering the process of the terminal guidance of missile, the following linear ISM manifold is introduced

$$s_1 = x_2 - x_2(0) + \int_0^t (k_1 x_1 + k_2 x_2) dt, \quad t \geq 0, \quad (8)$$

where $k_1 > 0$, $k_2 > 0$ are parameters to be designed. Note that it is $s_1(0) = 0$ at $t = 0$, that means the LOS angle error q_e and the LOS angular rate error \dot{q}_e occur on the ISM manifold (8) starting from the initial time instance.

Theorem 1. Consider guidance system (1)–(4) and (6). If Assumptions 1 and 2 are satisfied, and the switch gain satisfies $\varepsilon > \sqrt{d_1^2 + d_2^2}$, the following linear ISM guidance law based on the linear ISM manifold (8)

$$a_m = \frac{-2\dot{r}x_2 + k_1 r x_1 + k_2 r x_2 + \dot{V}_m \sin(q - \theta_m) + \varepsilon \text{sign}(s_1)}{\cos(q - \theta_m)} \quad (9)$$

can guarantee that the LOS angular rate \dot{q} asymptotically converges to zero, and the LOS angle q asymptotically converges to the desired LOS angle q_d .

Note. $\text{sign}(\cdot)$ in linear ISM guidance law (9) denotes the standard signum function.

Proof of Theorem 1. Differentiation of Eq. (8) with respect to time yields

$$\dot{s}_1 = \dot{x}_2 + k_2 x_2 + k_1 x_1. \quad (10)$$

Substitution of Eqs. (6) and (9) into Eq. (10) yields

$$\begin{aligned} \dot{s}_1 &= \frac{1}{r} [-2\dot{r}x_2 - a_m \cos(q - \theta_m) + \dot{V}_m \sin(q - \theta_m) \\ &\quad + a_t \cos(q - \theta_t) - \dot{V}_t \sin(q - \theta_t)] + k_2 x_2 + k_1 x_1 \\ &= \frac{1}{r} [a_t \cos(q - \theta_t) - \dot{V}_t \sin(q - \theta_t) - \varepsilon \operatorname{sign}(s)]. \end{aligned} \quad (11)$$

Consider the final Lyapunov function

$$V_1 = \frac{1}{2} s_1^2. \quad (12)$$

The derivative of Eq. (12) along the trajectories of Eq. (11), yields

$$\begin{aligned} \dot{V}_1 &= s_1 \dot{s}_1 = \frac{s_1}{r} [a_t \cos(q - \theta_t) - \dot{V}_t \sin(q - \theta_t) - \varepsilon \operatorname{sign}(s_1)] \\ &= \frac{s_1}{r} \left[\sqrt{a_t^2 + \dot{V}_t^2} \left(\frac{a_t}{\sqrt{a_t^2 + \dot{V}_t^2}} \cos(q - \theta_t) \right. \right. \\ &\quad \left. \left. - \frac{\dot{V}_t}{\sqrt{a_t^2 + \dot{V}_t^2}} \sin(q - \theta_t) \right) - \varepsilon \operatorname{sign}(s_1) \right] \\ &= \frac{s_1}{r} \left[\sqrt{a_t^2 + \dot{V}_t^2} \cos(q - \theta_t + \phi) - \varepsilon \operatorname{sign}(s_1) \right] \\ &\leq -\frac{\varepsilon - \sqrt{d_1^2 + d_2^2}}{r} |s_1|, \end{aligned} \quad (13)$$

where $\cos \phi = \frac{a_t}{\sqrt{a_t^2 + \dot{V}_t^2}}$ and $\sin \phi = \frac{\dot{V}_t}{\sqrt{a_t^2 + \dot{V}_t^2}}$.

According to Assumption 2, during the time horizon of the guidance process, it has

$$\dot{r}(t) < 0, \quad 0 < r(t) < r(0), \quad t > 0. \quad (14)$$

Combining Eqs. (13) and (14), it obtains

$$\dot{V}_1 \leq -\frac{\varepsilon - \sqrt{d_1^2 + d_2^2}}{r(0)} |s_1| = -\frac{\sqrt{2}(\varepsilon - \sqrt{d_1^2 + d_2^2})}{r(0)} V_1^{1/2}. \quad (15)$$

According to Lemma 1, Eq. (15) indicates that the LOS angle error q_e converges to the linear ISM manifold (8) in finite-time. Assume that the LOS angle error q_e reaches the manifold at t_1 . According to Lemma 1, it can obtain

$$t_1 \leq \frac{\sqrt{2}r(0)}{\varepsilon - \sqrt{d_1^2 + d_2^2}} V_1^{1/2}(0) = \frac{r(0)}{\varepsilon - \sqrt{d_1^2 + d_2^2}} |s_1(0)| = 0. \quad (16)$$

Eq. (16) demonstrates that, even if there exists the external disturbance, the LOS angle error q_e and the LOS angular rate error \dot{q}_e occur on the ISM manifold (8) starting from the initial time instance.

In the end, we prove the LOS angle error q_e can asymptotically converge to zero. Because of the system states q_e and \dot{q}_e occurring on linear ISM manifold, it follows from Eq. (8) that

$$s_1 = x_2 - x_2(0) + \int_0^t (k_1 x_1 + k_2 x_2) dt = 0, \quad t \geq t_1 = 0, \quad (17)$$

which implies that $\dot{s}_1 = 0$, i.e.,

$$\dot{s}_1 = \dot{x}_2 + k_2 x_2 + k_1 x_1 = \ddot{q}_e + k_2 \dot{q}_e + k_1 q_e = 0. \quad (18)$$

According to the Routh Criterion, the condition $k_1 > 0$, $k_2 > 0$ is sufficient to guarantee that the LOS angle error q_e asymptotically converges to zero with infinite convergence time. Hence, the LOS angular rate error \dot{q}_e asymptotically converges to zero. Note that $\dot{q}_e = \dot{q} - \dot{q}_d = \dot{q} - \dot{q}_d$ and $q_e = q - q_d$, which demonstrate that the LOS angular rate \dot{q} will asymptotically converge to zero and the LOS angle q will asymptotically converge to the desired LOS angle q_d . The proof is finished. \square

3.2. Guidance law based on nonlinear integral sliding mode control

The proposed linear ISM guidance law in Section 3.1 can guarantee the LOS angular rate and the LOS angle asymptotical convergence with infinite settling time. To further improve the convergence characteristics of LOS angle and LOS angular rate, one possible method is to introduce a nonlinear control strategy. In this section, a finite-time control strategy is introduced. Firstly, a nonlinear ISM manifold based on a homogeneity controller for integrator system is introduced. And then, a nonlinear ISM guidance law based on the nonlinear ISM manifold is designed. Finally, it is proved that the obtained guidance law can ensure LOS angle and LOS angular rate the finite-time convergence characteristics. Before the design of finite-time convergence guidance laws, we need some results on the finite-time stability for nonlinear systems.

Lemma 2. (See [3].) For $\alpha \in (0, 1)$, the feedback law

$$u = \phi(\xi_1, \xi_2) = -k_1 |\xi_1|^\alpha \operatorname{sign}(\xi_1) - k_2 |\xi_2|^{\frac{2\alpha}{\alpha+1}} \operatorname{sign}(\xi_2) \quad (19)$$

renders the origin finite-time stable for the double integrator system

$$\dot{\xi}_1 = \xi_2, \quad \dot{\xi}_2 = u, \quad (20)$$

where $k_1 > 0$, $k_2 > 0$.

Remark 1. (See [14].) The origin of system (20) is finite-time stable. There exists an explicit finite-time Lyapunov function

$$\begin{aligned} V_0 &= \frac{2 + 2k_0^{(3+\alpha)/2}}{3 + \alpha} |\xi_1|^{(3+\alpha)/2} \\ &\quad + \frac{(1 + \alpha)^2}{5\alpha + 1} |\xi_2|^{(3+\alpha)/(1+\alpha)} + k_0 \xi_1 \xi_2, \end{aligned} \quad (21)$$

where $k_0 = (k_1/k_2)^{1/\alpha} > 0$ with k_2 sufficiently large, and then it can obtain that there is a K such that

$$\dot{V}_0 \leq -K V_0^{(2+2\alpha)/(3+\alpha)}. \quad (22)$$

From Lemma 1, this inequality leads to the convergence time estimation given by

$$T_0 \leq \frac{(3 + \alpha) V_0^{(1-\alpha)/(3+\alpha)}(\xi_0)}{K(1 - \alpha)}, \quad (23)$$

where ξ_0 is the initial state of the system.

Similarly to the design method of linear ISM guidance law, a nonlinear ISM manifold inspired by Zong [49] is introduced,

$$\begin{aligned} s_2 &= x_2 - x_2(0) + \int_0^t [k_1 |x_1|^\alpha \operatorname{sign}(x_1) + k_2 |x_2|^{\frac{2\alpha}{\alpha+1}} \operatorname{sign}(x_2)] dt, \\ t &\geq 0, \end{aligned} \quad (24)$$

where $k_1 > 0$, $k_2 > 0$, $0 < \alpha < 1$ are parameters to be designed. Note that it is also $s_2(0) = 0$ at $t = 0$, that means the LOS angle error q_e and the LOS angular rate error \dot{q}_e occur on the nonlinear ISM manifold (24) starting from the initial time instance.

Theorem 2. Consider guidance system (1)–(4) and (6). If Assumptions 1 and 2 are satisfied, and the switch gain satisfies $\varepsilon > \sqrt{d_1^2 + d_2^2}$, the following nonlinear ISM guidance law based on the nonlinear ISM manifold (24)

$$a_m = (-2\dot{x}_2 + k_1 r |x_1|^\alpha \text{sign}(x_1) + k_2 r |x_2|^{\frac{2\alpha}{\alpha+1}} \text{sign}(x_2) + \dot{V}_m \sin(q - \theta_m) + \varepsilon \text{sign}(s_2)) (\cos(q - \theta_m))^{-1} \quad (25)$$

can guarantee that the LOS angular rate \dot{q} converges to zero in finite time, and the LOS angle q converges to the desired LOS angle q_d in finite time.

Proof. Differentiation of Eq. (24) with respect to time yields

$$\dot{s}_2 = \dot{x}_2 + k_2 |x_2|^{\frac{2\alpha}{\alpha+1}} \text{sign}(x_2) + k_1 |x_1|^\alpha \text{sign}(x_1). \quad (26)$$

Substitution of Eqs. (6) and (21) into Eq. (22) yields

$$\begin{aligned} \dot{s}_2 &= \frac{1}{r} [-2\dot{x}_2 - a_m \cos(q - \theta_m) + \dot{V}_m \sin(q - \theta_m) \\ &\quad + a_t \cos(q - \theta_t) - \dot{V}_t \sin(q - \theta_t)] \\ &\quad + k_2 |x_2|^{\frac{2\alpha}{\alpha+1}} \text{sign}(x_2) + k_1 |x_1|^\alpha \text{sign}(x_1) \\ &= \frac{1}{r} [a_t \cos(q - \theta_t) - \dot{V}_t \sin(q - \theta_t) - \varepsilon \text{sign}(s_2)]. \end{aligned} \quad (27)$$

Similarly to the proof of Theorem 1, the final Lyapunov function is chosen as $V_2 = \frac{1}{2}s_2^2$. The derivative of Lyapunov function along the trajectory of Eq. (27), yields

$$\dot{V}_2 = s_2 \dot{s}_2 \leq -\frac{\sqrt{2}(\varepsilon - \sqrt{d_1^2 + d_2^2})}{r(0)} V_2^{1/2}. \quad (28)$$

According to Lemma 1, Eq. (28) demonstrates that the LOS angle error q_e converges to the nonlinear ISM manifold (24) in finite time. Assume that the LOS angle error reaches the manifold at t_2 . According to Lemma 1, it can obtain

$$t_2 \leq \frac{r(0)}{\varepsilon - \sqrt{d_1^2 + d_2^2}} |s_2(0)| = 0. \quad (29)$$

Eq. (29) demonstrates that, even if there exists the external disturbance, the LOS angle error q_e and the LOS angular rate error \dot{q}_e occur on the nonlinear ISM manifold (24) starting from the initial time instance.

In the end, we prove that the LOS angle error q_e will converge to zero in finite time. Because of the system states q_e and \dot{q}_e occurring on nonlinear ISM manifold, it follows from Eq. (24) that

$$\begin{aligned} s_2 &= x_2 - x_2(0) \\ &\quad + \int_0^t [k_1 |x_1|^\alpha \text{sign}(x_1) + k_2 |x_2|^{\frac{2\alpha}{\alpha+1}} \text{sign}(x_2)] dt = 0, \\ t &\geq t_2 = 0, \end{aligned} \quad (30)$$

which implies that $\dot{s}_2 = 0$, i.e.,

$$\begin{aligned} \dot{s}_2 &= \dot{x}_2 + k_2 |x_2|^{\frac{2\alpha}{\alpha+1}} \text{sign}(x_2) + k_1 |x_1|^\alpha \text{sign}(x_1) \\ &= \ddot{q}_e + k_2 |\dot{q}_e|^{\frac{2\alpha}{\alpha+1}} \text{sign}(\dot{q}_e) + k_1 |q_e|^\alpha \text{sign}(q_e) = 0, \end{aligned} \quad (31)$$

where $k_1 > 0$, $k_2 > 0$, $0 < \alpha < 1$.

According to Lemma 2, system (31) is finite-time stable. Therefore, the LOS angle error q_e and the LOS angular rate error \dot{q}_e converge to zero in finite time. Note that $\dot{q}_e = \dot{q} - \dot{q}_d = \dot{q}$ and $q_e = q - q_d$, which mean the LOS angular rate \dot{q} will converge to zero in finite time and the LOS angle q will converge to the desired LOS angle q_d in finite time.

According to Remark 1, the convergence time estimation is given by

$$T \leq \frac{(3 + \alpha)V_0^{(1-\alpha)/(3+\alpha)}(x_0)}{K(1 - \alpha)}, \quad (32)$$

where $K > 0$, x_0 is the initial state of the system.

The proof is finished. \square

Remark 2. On the one hand, due to existence of function $\varepsilon \text{sign}(s)$, both ISM guidance laws (9) and (25) are discontinuous, thus the system may generate chattering problem. On the other hand, to guarantee that the guidance system has a good disturbance rejection performance, the switch gain ε needs to be chosen larger than the bound of the target acceleration. However, if there is no precise information on the bound of the target acceleration, the switching gain will be taken to be large enough. This inaccuracy worsens the chattering caused by sliding mode control. Note that, the functions $k_1 r |x_1|^\alpha \text{sign}(x_1)$ and $k_2 r |x_2|^{\frac{2\alpha}{\alpha+1}} \text{sign}(x_2)$ in guidance law (25) are continuous, which do not generate chattering. To handle chattering phenomenon, one of the solution is to employ a saturation function $\varepsilon s/(|s| + \delta)$, where δ is a very small positive number, to substitute the signum function $\varepsilon \text{sign}(s)$ in guidance laws (9) and (25). This method can remove the chattering phenomenon. However, to do this, the disturbance rejection performance is sacrificed to some extent. Another efficient method for suppressing the chattering phenomenon is to employ a NDOB to estimate the target acceleration. This estimate of target acceleration will be employed as a compensation item in ISM guidance laws. After disturbance compensation based on NDOB, the switching gain only needs to be larger than the bound of the disturbance compensation error which is usually much smaller than that of the target acceleration. Hence, the chattering phenomenon will be reduced while the disturbance rejection performance of the guidance system can be maintained.

4. Composite guidance law based on nonlinear disturbance observer

To handle the chattering phenomenon and maintain the performance of the guidance system, we will integrate the ISM guidance laws and NDOB to give composite guidance control schemes. First of all, by designing a non-smooth NDOB, the target acceleration can be estimated in finite time. This estimated target acceleration will be employed as a feedforward compensation item in the ISM guidance laws (9) and (25). Then, composite guidance control schemes can be obtained.

4.1. Nonlinear disturbance observer

Consider single-input single-output (SISO) dynamics

$$\dot{\sigma} = g(t) + u, \quad (33)$$

where $\sigma \in R$, $u \in R$ is a control input, $g(t)$ is an uncertain sufficiently smooth function and satisfies $\dot{g}(t) \leq L$, $L > 0$ is Lipschitz constant. The control function $u(t)$ is Lebesgue measurable. Eq. (33) is understood in the Filippov sense [18], which means in particular that $\sigma(t)$ is an absolutely continuous function defined for any $t \geq 0$. By a simple modification of Theorem 6 in [18], we have the following result.

Lemma 3. (See [18].) Consider the following NDOB:

$$\begin{cases} \dot{z}_0 = v_0 + u, & v_0 = -\lambda_0 |z_0 - \sigma|^{2/3} \text{sign}(z_0 - \sigma) + z_1, \\ \dot{z}_1 = v_1, & v_1 = -\lambda_1 |z_1 - v_0|^{1/2} \text{sign}(z_1 - v_0) + z_2, \\ \dot{z}_2 = -\lambda_2 \text{sign}(z_2 - v_1), \end{cases} \quad (34)$$

where, $\lambda_i > 0$, $i = 0, 1, 2$ are sufficiently large, then it can obtain that z_1 converges to $g(t)$ in finite time.

Remark 3. It is proved in [18] that in order to obtain the finite-time convergence of the observation errors, the parameters λ_i , $i = 0, 1, 2$ should be chosen large enough. However, according to [29], the simulation shows that $\lambda_0 = 2L^{1/3}$, $\lambda_1 = 1.5L^{1/2}$ and $\lambda_3 = 1.1L$ are enough to ensure the stability of observation error.

According to guidance system (6), define $w_q = a_t \cos(q - \theta_t) - \dot{V}_t \sin(q - \theta_t)$ which is the target acceleration component normal to the LOS. The design object is to develop a NDOB such that the estimated target acceleration \hat{w}_q can converge to actual target acceleration w_q in finite time.

And, define $d(t) = \frac{1}{r} w_q$ as lumped disturbance and $\hat{d}(t)$ as the estimated disturbance. Let $G(t) = g(t) + d(t)$, $g(t) = \frac{1}{r}(-2\dot{r}x_2 + \dot{V}_m \sin(q - \theta_m))$ and $B = -\frac{1}{r} \cos(q - \theta_m)$. Guidance system (6) can be written as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = G(t) + Ba_m. \end{cases} \quad (35)$$

The derivatives of functions $g(t)$ and $d(t)$ are $\dot{g}(t) = \frac{1}{r^2}[2\dot{r}^2x_2 - 2r\ddot{r}x_2 - 2r\dot{r}\dot{x}_2 + (r\ddot{V}_m - \dot{r}\dot{V}_m)\sin(q - \theta_m) + r\dot{V}_m(\dot{q} - \dot{\theta}_m)\cos(q - \theta_m)]$, $\dot{d}(t) = \frac{1}{r^2}[(r\dot{a}_t - \dot{r}a_t)\cos(q - \theta_t) + (\dot{r}\dot{V}_t - r\ddot{V}_t)\sin(q - \theta_t) - r(\dot{q} - \dot{\theta}_t) \times (\dot{V}_t \cos(q - \theta_t) + a_t \sin(q - \theta_t))]$, respectively.

Remark 4. If the target acceleration a_t and \dot{V}_t are differentiable, then the function $g(t)$ and $d(t)$ are differentiable except for $r = 0$. According to Assumption 2, the functions $g(t)$ and $d(t)$ are differentiable and the $\dot{g}(t)$ and $\dot{d}(t)$ are continuous everywhere until hit-to-kill happens. Since the function $G(t) = g(t) + d(t)$ is continuously differentiable for any $r \geq r^0$, its derivative $\dot{G}(t)$ has a Lipschitz constant L .

For guidance system (6), according to Lemma 3, the lump disturbance $d(t)$ can be estimated by the following NDOB (36)

$$\begin{cases} \dot{z}_0 = v_0 + Ba_m + g(t), \\ v_0 = -\lambda_0|z_0 - x_2|^{2/3} \text{sign}(z_0 - x_2) + z_1, \\ \dot{z}_1 = v_1, \quad v_1 = -\lambda_1|z_1 - v_0|^{1/2} \text{sign}(z_1 - v_0) + z_2, \\ \dot{z}_2 = -\lambda_2 \text{sign}(z_2 - v_1), \\ \hat{d}(t) = z_1. \end{cases} \quad (36)$$

According to Lemma 3 and Remark 3, it can obtain that the estimated lumped disturbance $\hat{d}(t)$ converges to $d(t)$ in finite time, i.e., the estimated target acceleration $\hat{w}_q(t) = r\hat{d}(t)$ converges to real target acceleration $w_q(t)$ in finite time.

4.2. Composite guidance law

Before giving the composite guidance law, the following assumption is presented on the basis of Lemma 3.

Assumption 3. The target acceleration estimation error is bounded and there exists a constant ϱ , such that

$$|w_q(t) - \hat{w}_q(t)| \leq \varrho. \quad (37)$$

Theorem 3. Consider guidance system (1)–(4) and (6). If Assumptions 2 and 3 are satisfied, and the switch gain satisfies $\eta > \varrho$, the following composite linear guidance law combining the linear ISM guidance law (9) with NDOB (36)

$$\begin{aligned} a_m = & (-2\dot{r}x_2 + k_1rx_1 + k_2rx_2 + \dot{V}_m \sin(q - \theta_m) + \eta \text{sign}(s_1) \\ & + \hat{w}_q(t))(\cos(q - \theta_m))^{-1} \end{aligned} \quad (38)$$

can guarantee that the LOS angular rate \dot{q} asymptotically converges to zero, and the LOS angle q asymptotically converges to the desired LOS angle q_d .

Proof. Substituting Eqs. (6) and (38) into Eq. (10), yields

$$\dot{s}_1 = \frac{1}{r}[w_q(t) - \hat{w}_q(t) - \eta \text{sign}(s_1)]. \quad (39)$$

Similarly to the proof of Theorem 1, the final Lyapunov function is chosen as $V_3 = \frac{1}{2}s_1^2$. The derivative of Lyapunov function along the trajectory of Eq. (39), yields

$$\begin{aligned} \dot{V}_3 = s_1\dot{s}_1 &= \frac{s_1}{r}[w_q(t) - \hat{w}_q(t) - \eta \text{sign}(s_1)] \leq -\frac{\eta - \varrho}{r}|s_1| \\ &\leq -\frac{\sqrt{2}(\eta - \varrho)}{r(0)}V_3^{1/2}. \end{aligned} \quad (40)$$

According to Lemma 1, Eq. (40) demonstrates that the LOS angle error q_e converges to the linear ISM manifold Eq. (8) in finite time.

The rest procedure is identical with that of Theorem 1, which is omitted here. Similarly to the proof of Theorem 1, we obtain that the LOS angular rate \dot{q} will asymptotically converge to zero and the LOS angle q will asymptotically converge to the desired LOS angle q_d . The proof is finished. \square

Theorem 4. Consider guidance system (1)–(4) and (6). If Assumption 2 and 3 are satisfied, and the switch gain satisfies $\eta > \varrho$, the following nonlinear composite guidance law combining the nonlinear ISM guidance law (25) with NDOB (36)

$$\begin{aligned} a_m = & (-2\dot{r}x_2 + k_1r|x_1|^\alpha \text{sign}(x_1) + k_2r|x_2|^{\frac{2\alpha}{\alpha+1}} \text{sign}(x_2) \\ & + \dot{V}_m \sin(q - \theta_m) + \eta \text{sign}(s_2) + \hat{w}_q(t))(\cos(q - \theta_m))^{-1} \end{aligned} \quad (41)$$

can guarantee that the LOS angular rate \dot{q} converges to the origin in finite time, and the LOS angle q converges to the desired LOS angle q_d in finite time.

Proof. Substituting Eqs. (6) and (41) into Eq. (26), yields

$$\dot{s}_2 = \frac{1}{r}[w_q(t) - \hat{w}_q(t) - \eta \text{sign}(s_2)]. \quad (42)$$

Similarly with the proof of Theorem 2, the final Lyapunov function is chosen as $V_4 = \frac{1}{2}s_2^2$, also. The derivative of Lyapunov function along the trajectory of Eq. (42), yields

$$\dot{V}_4 = s_2\dot{s}_2 \leq -\frac{\sqrt{2}(\eta - \varrho)}{r(0)}V_4^{1/2}. \quad (43)$$

According to Lemma 1, Eq. (43) demonstrates that the LOS angle error q_e converges to the nonlinear ISM manifold Eq. (24) in finite time.

The rest procedure is identical with that of Theorem 2, which is omitted here. Similarly to the proof of Theorem 2, we obtain that the LOS angular rate \dot{q} will converge to zero in finite time and the LOS angle q will converge to the desired LOS angle q_d in finite time. The proof is finished. \square

Remark 5. The differences between Theorems 1, 2 and Theorems 3, 4 are that feedforward compensation terms \hat{w}_q are introduced into controller in Theorems 3 and 4. When the parameters of the NDOB are appropriately chosen, w_q will be well estimated. Usually

the bound of disturbance compensation errors can be less than the disturbances. In this case, the composite guidance laws (38) and (41) may select a smaller value for the switching gain η without sacrificing disturbance rejection performance, which helps to reduce large chattering caused by high gain.

Remark 6. The composite guidance law and NDOB are designed separately. The control parameters and the NDOB parameters can be also chosen separately. The NDOB parameters λ_i , $i = 0, 1, 2$ can be obtained according to Remark 3. For the Lipschitz constant L , when L is chosen from small to large value, the disturbance estimation dynamics response becomes faster. However, when its value is large enough, the overshoot appears accordingly. The parameters L should be selected suitably considering trade off between response speed and overshoot. The guidance law parameters $k_1 > 0$, $k_2 > 0$, $0 < \alpha < 1$, $\varepsilon > 0$ and $\eta > 0$ directly influence the closed loop performance. The control parameters k_1 , k_2 can be obtained by pole assignment method. From the stability analysis of Theorems 1 to 4, switch gain ε should be larger than the bound of lumped disturbances for ISM guidance laws (9), (25), and switch gain η should be large than the bound of disturbance compensation error for composite guidance laws (38), (41). Usually, when the parameter α is chosen smaller, the convergence rate of states will be faster.

5. Simulation results

In this section, the numerical examples are performed to illustrate the performance of all proposed guidance laws (9), (25), (38), (41). The initial states of guidance system are chosen as follows. The initial relative distance between missile and target is $r_0 = 5000$ m. The initial LOS angle is $q_0 = 30$ deg. The desired LOS angle is $q_d = 20$ deg. The initial missile velocity is $V_{m0} = 600$ m/s with initial flight-path angle $\theta_{m0} = 60$ deg. In the simulation, the missile longitudinal acceleration is $\dot{V}_m = 0$ m/s², i.e., the missile velocity V_m is a constant. The initial target velocity is $V_{t0} = 300$ m/s with flight-path angle $\theta_{t0} = 0$ deg. The acceleration of gravity is $g = 9.8$ m/s². The initial states of NDOB (36) are chosen as $z_0 = [0, 0.5, 0]^T$.

In order to demonstrate the effectiveness of the proposed guidance laws, the PN guidance law is chosen for comparison. The PN guidance law is taken as

$$n_c = -N\dot{r}\dot{q}, \quad (44)$$

where N is a unitless coefficient of PN guidance law. To have a fair comparison for the simulation results, the acceleration command is limited not to exceed 50 g. The parameter N in PN guidance law is chosen as $N = 5$. According to Remark 6, the parameters k_1 and k_2 of all proposed guidance laws are obtained by pole placement method. The parameters of the linear ISM guidance law (9) and composite linear guidance law (38) are selected as $k_1 = 1$ and $k_2 = 2$. And, the parameters of the nonlinear ISM guidance law (25) and composite nonlinear guidance law (41) are selected as $k_1 = 0.5$, $k_2 = 1$ and $\alpha = 0.3$. The switch gain ε of ISM guidance laws (9) and (25) is selected as $\varepsilon = 70$. And, the switch gain η of composite guidance laws (38) and (41) is selected as $\eta = 0.1$. The Lipschitz constant L of NDOB is chosen as $L = 50$. According to Remark 3, the gains λ_0 , λ_1 and λ_2 of NDOB can be chosen as $\lambda_0 = 7.368$, $\lambda_1 = 10.607$, $\lambda_2 = 55$. The following cases are simulated.

Case 1. It is supposed that the target acceleration is chosen as

$$a_t(t) = \begin{cases} 5g, & t \leq 7 \\ -10g, & t > 7, \end{cases} \quad \dot{V}_t = \begin{cases} 0, & t \leq 10 \\ 40, & t > 10. \end{cases} \quad (45)$$

The response of LOS angle, LOS angular rate, estimated disturbance, missile acceleration command, sliding mode manifold and interception geometry are shown in Figs. 2(a), (b), (c), (d), (e) and (f), respectively. The miss distances and interception time are shown in Table 1.

Note. The abbreviations in Fig. 2 and Table 1, i.e., PNGL, LISMGL, NISMGL, CLGL and CNGL, denote the PN guidance law (44), linear ISM guidance law (9), nonlinear ISM guidance law (25), composite linear guidance law (38) and composite nonlinear guidance law (41), respectively. In the following cases, the abbreviations have the same meanings.

From Table 1, it is clear that four proposed guidance laws (9), (25), (38), (41) can guarantee the miss distance of guidance system less than 0.01 m in spite of the target acceleration with significant changes. According to Ref. [30], it indicates that the missile can intercept the large maneuvering target by hit-to-kill guidance strategy. However, the PN guidance law makes the missile has a large miss distance 353.64 m, which means the missile will miss the target. Therefore, the four proposed guidance laws are available to intercept the large maneuvering target, while the PN guidance law cannot.

As shown in Fig. 2(a), it can be seen that the four proposed guidance laws can guarantee the LOS angle converges to the desired angle 20 deg. Fig. 2(b) shows that all proposed guidance laws can guarantee the LOS angular rate converges to zero. But, the PN guidance law cannot guarantee the convergence of LOS angle and LOS angular rate. From Figs. 2(a') and 2(b'), it is clear that the nonlinear guidance laws (25), (41) guarantee LOS angle and LOS angular rate more rapidly convergence rate than that of linear guidance laws (9), (38). And, the nonlinear guidance laws (25), (41) can guarantee the LOS angle and LOS angular rate finite-time convergence. Fig. 2(c) shows that the designed NDOB can observe the target acceleration. It is clear from Fig. 2(c') that the estimated target acceleration \hat{w}_q converges to the real target acceleration w_q in finite time. The control input signals of all proposed guidance laws are shown in Figs. 2(d, d'). It is clear that, for ISM guidance laws (9), (25), the guidance commands cause the chattering phenomenon. That is because, to suppress the target acceleration, the switch gain ε is chosen larger than the bound of target acceleration. But, for composite guidance laws (38), (41), the undesired chattering is reduced effectively. That is because that, under the influence of NDOB, the estimated target acceleration \hat{w}_q can be a good estimate of actual target and the upper bound of $\hat{w}_q - w_q$ can be less than w_q . In this case, the switching gain η required usually is much smaller than ε . Thus, the discontinuous terms of the composite guidance laws (9), (25) can be much smaller and the chattering will be reduced. Figs. 2(e, e') depict the performance of sliding mode manifold. It can be seen that the sliding mode is stable in spite of the target acceleration. The interception geometries are shown in Figs. 2(f, f'), which shows that four proposed guidance laws can accomplish the interception, while the PN guidance law cannot.

In order to demonstrate that the control strategy can also work well when the autopilot dynamics is considered, the following case is simulated.

Case 2. On the basis of Case 1, the autopilot dynamics is considered in this case. During the design process of guidance laws, the autopilot dynamics is regarded as the ideal dynamics. However, the autopilot dynamics exists in the actual missile system. Therefore, it is necessary to observe the interference caused by autopilot dynamics. In this case, a second-order transfer function of missile autopilot dynamics [17] is selected as

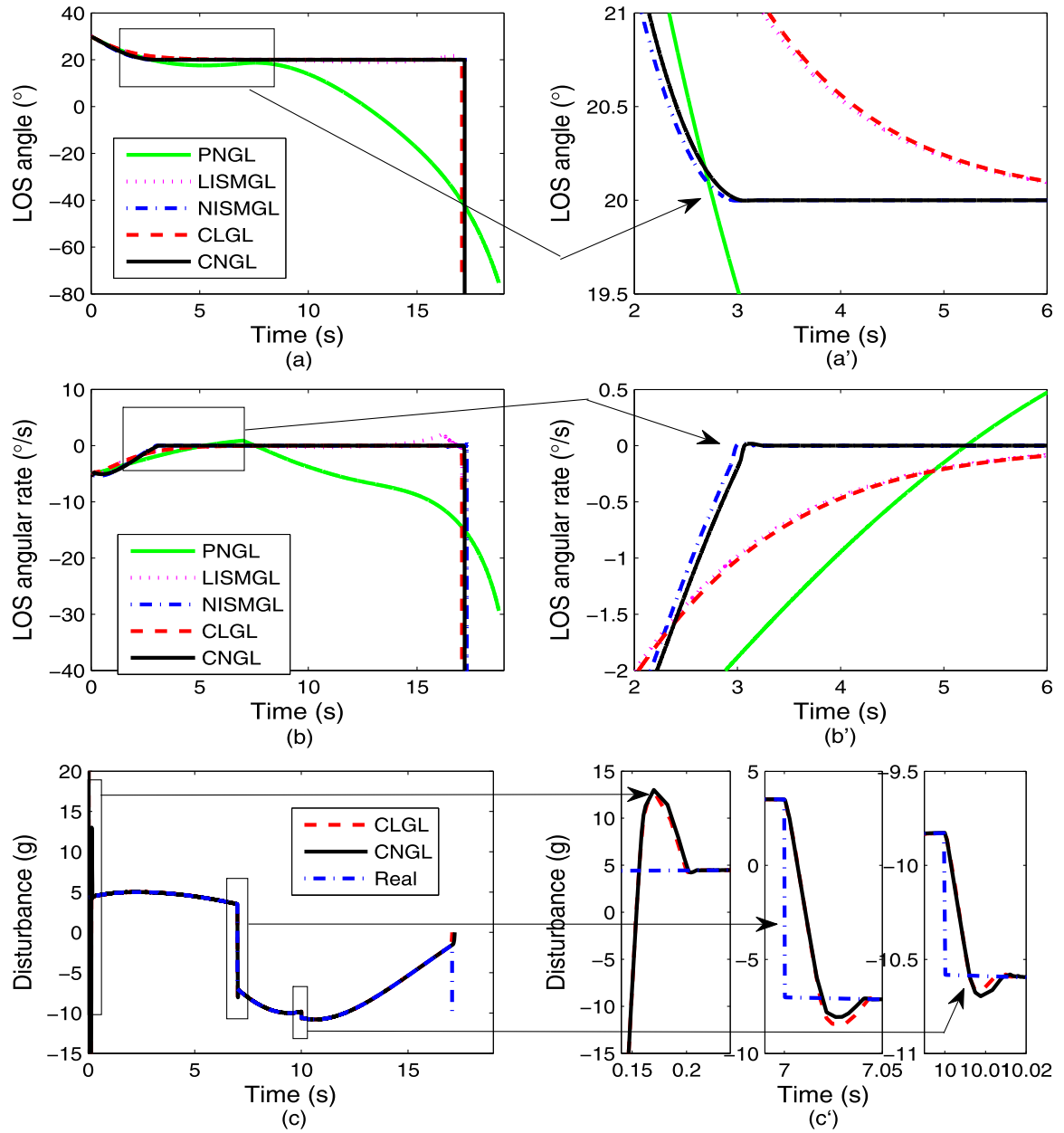


Fig. 2. Responses under five guidance laws of Case 1. (a) LOS angle. (b) LOS angular rate. (c) Real and estimated disturbances. (d) Missile acceleration command. (e) Sliding mode manifold. (f) Interception geometry.

$$\frac{a_{mo}}{a_m} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (46)$$

where, a_{mo} is the actual output acceleration of missile autopilot, ζ is the damping coefficient, and ω_n is the natural frequency, respectively. In the simulation process, the parameters are chosen as $\omega_n = 10$, $\zeta = 0.5$, respectively. The response of LOS angle, LOS angular rate, estimated disturbance, missile acceleration command, sliding mode dynamic and interception geometry are shown in Figs. 3(a), (b), (c), (d), (e) and (f), respectively. The miss distances and interception time are shown in Table 2.

Similarly to Case 1, it can be observed from Table 2 that four proposed guidance laws (9), (25), (38), (41) can guarantee the miss distance of guidance system less than 0.01 m while the autopilot dynamics is considered, which means that the missile can accomplish hit-to-kill guidance strategy. The PN guidance law makes the

Table 1
Miss distances and interception time of Case 1.

Guidance law	Miss distance (m)	Interception time (s)
PNGL	353.64	18.765
LISMGL	6.79×10^{-12}	17.077
NISMGL	1.65×10^{-12}	17.293
CLGL	3.82×10^{-8}	17.071
CNGL	5.07×10^{-9}	17.191

missile has a large miss distance 379.37 m, which means the missile will miss the target. Therefore, the four proposed guidance laws are available to intercept large maneuvering target in the existence of autopilot dynamics.

Fig. 3(a) depicts that the four proposed guidance laws can guarantee that the LOS angle converges to the desired angle 20 deg. As shown in Fig. 3(b), it is clear that proposed guidance laws (25), (41) can guarantee that the LOS angular rates converge to zero.

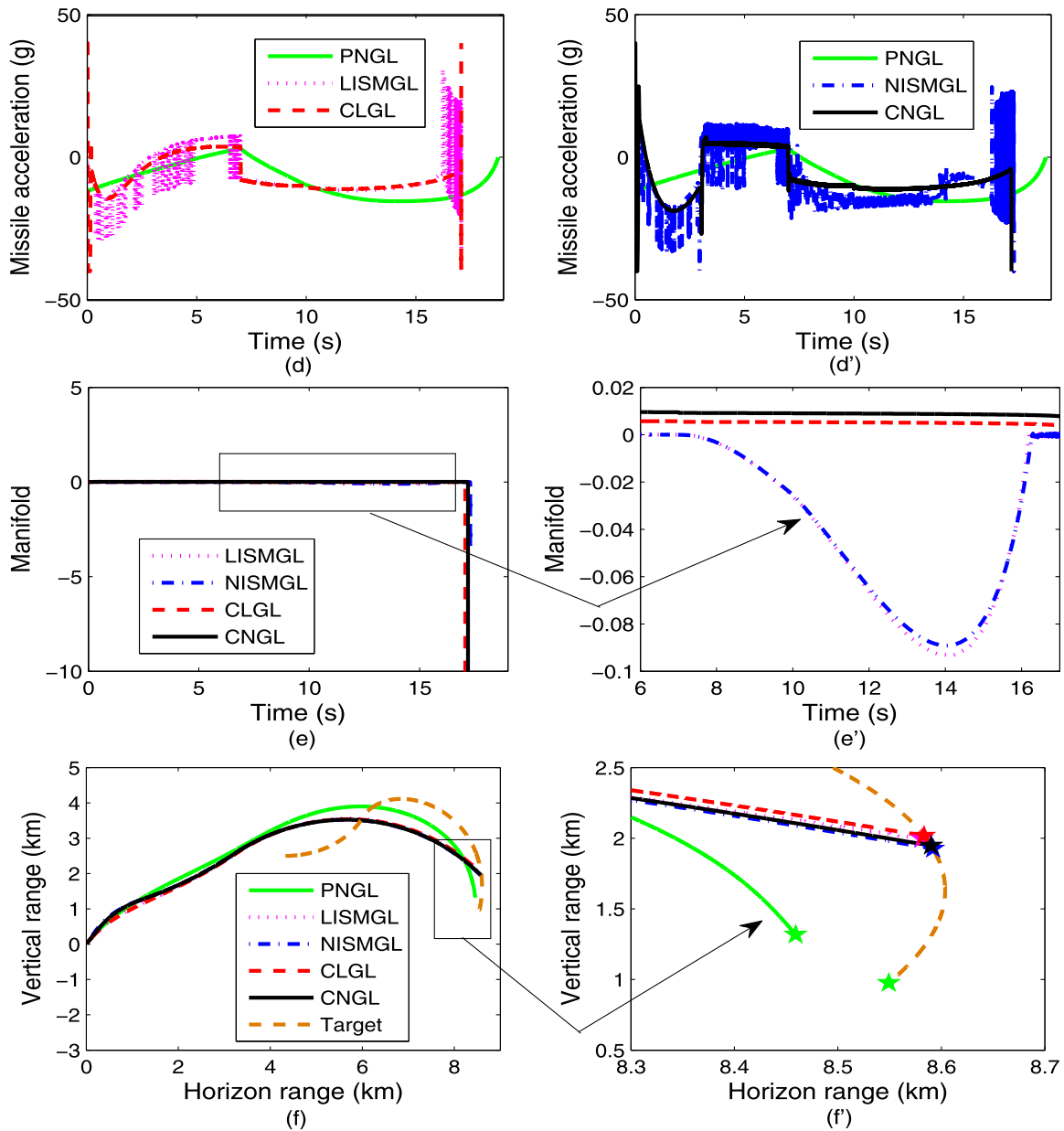


Fig. 2. (continued)

Table 2
Miss distances and interception time of Case 2.

Guidance law	Miss distance (m)	Interception time (s)
PNGL	379.37	18.787
LISMGL	2.36×10^{-2}	17.287
NISMGL	1.04×10^{-2}	17.225
CLGL	2.12×10^{-2}	17.057
CNGL	4.18×10^{-2}	17.116

And, it is obvious that the nonlinear guidance laws guarantee LOS angle and LOS angular rate have the finite-time convergence rate, which is more rapidly than that of linear guidance laws. Figs. 3(c, c') show that the estimated target acceleration converges to the real target acceleration in finite time. The control input signals of four proposed guidance laws are shown in Figs. 3(d, d'). By comparing Figs. 2(d, d') and Figs. 3(d, d'), one can observe that, when the autopilot dynamics is not considered, the ISM guidance laws (9), (25) cause heavily chattering phenomenon. But, when

the autopilot dynamics is considered, the chattering phenomenon will be moderated. That is because that the second-order transfer function acts as a low-pass filter on the command. It will reduce the chattering phenomenon caused by discontinuous term. It is clear from Fig. 3(e), that the sliding mode manifolds are affected by the autopilot dynamics. The interception geometries are shown in Fig. 3(f), which shows that four proposed guidance laws accomplish the interception while the PN guidance law cannot.

In conclusion, for missiles intercepting large maneuvering targets with impact angle constraint, four proposed guidance laws (9), (25), (38), (41) work effectively and are robust to target acceleration and autopilot dynamics. The proposed linear guidance laws (9), (38) guarantee the LOS angle q and LOS angular rate \dot{q} asymptotical convergence property. The proposed nonlinear guidance laws (25), (41) employ the finite-time control strategy, which guarantee the LOS angle q and LOS angular rate \dot{q} finite-time convergence property. Four proposed guidance laws

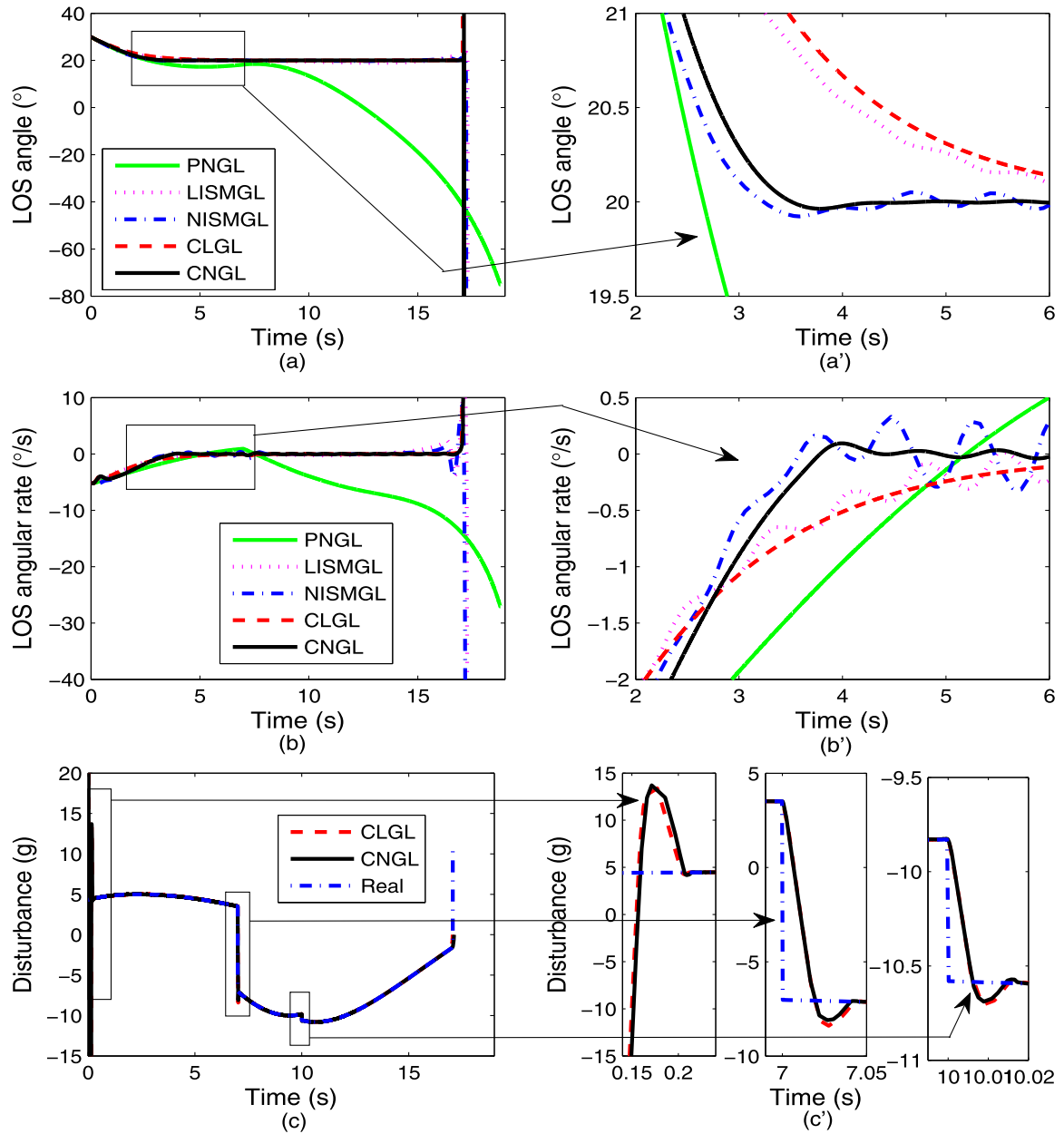


Fig. 3. Responses under five guidance laws of Case 2. (a) LOS angle. (b) LOS angular rate. (c) Real and estimated disturbances. (d) Missile acceleration command. (e) Sliding mode manifold. (f) Interception geometry.

can ensure that the missiles hit the target with zero miss disturbance. The NDOB can work effectively to estimate the target acceleration. The estimated target acceleration converges to the real target acceleration in finite time. After disturbance compensation based on NDOB, composite guidance laws (38), (41) have the ability to reduce the chattering and remain the performance of that which ISM guidance laws (9), (25) have. Thus, the proposed guidance laws (9), (25), (38), (41) achieve the design objective.

6. Conclusion

In this paper, based on ISMC and NDOB theory, guidance laws with impact angle constraint have been developed. All guidance laws are proposed to achieve the hit-to-kill guidance strategy in the presence of target maneuvers. Proposed linear guidance laws can make the LOS angle and LOS angular rate asymptot-

ical convergence. Proposed nonlinear guidance laws can guarantee the LOS angle and LOS angular rate finite-time convergence. With the help of NDOB by estimating the target acceleration, the composite guidance laws are proposed to reduce the chattering phenomenon which exists in ISM guidance laws and to remain the performance of that which ISM guidance laws have. Finally, simulation comparison results are provided to demonstrate the effectiveness of the presented methods.

Acknowledgements

This work is supported by Natural Science Foundation of China (61074013), Program for New Century Excellent Talents in University (NCET-10-0328), Science Foundation for Distinguished Young Scholars of Jiangsu Province (BK20130018), and National Aerospace Science Foundation of China (20090112002, 20100169002).

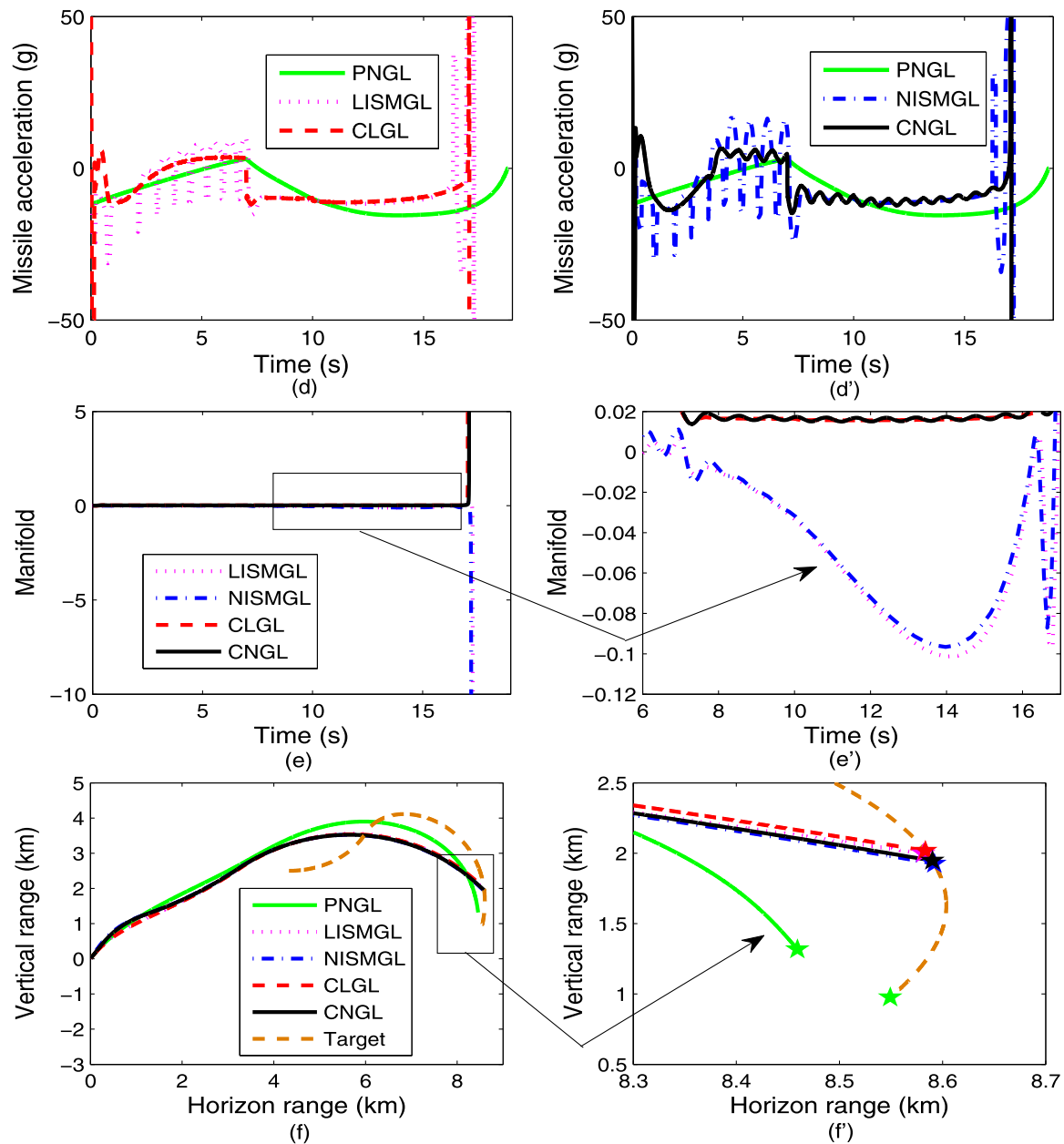


Fig. 3. (continued)

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